Optimal Ccombining Data for Improving Ocean Modeling

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LONG-TERM GOALS

The long range scientific objectives of the proposed research comprise: (1) developing rigorous approaches to optimal combining satellite and drifter data with an output of a regional circulation model for accurate estimating the upper ocean velocity field and mixing characteristics such as relative dispersion and finite size Lyapunov exponent, (2) constructing and comprehensive testing computationally efficient estimation algorithms based on alternative parameterizations of uncertainty, (3) processing real data in the Adriatic and Ligurian Sea (MREA coastal experiments) via new techniques

OBJECTIVES

The objectives for the first year of research were:
- Developing and verifying data fusion algorithms for optimal estimating surface velocities from tracer observations, Lagrangian data, and a circulation model output, based on the *fuzzy logic* approach [1,2].
- Carrying out a comprehensive error analysis via theoretical and Monte Carlo methods.
- Constructing and testing compatibility measures between data and model.
- Testing the developed fusion algorithms by 'twin' experiments via NCOM.
- Theoretical investigation of the absolute and relative dispersion (AD and RD) in the presence of shear flow, which would help in constructing realistic algorithms for estimating these characteristics from real data.
- Developing data fusion algorithms for estimating RD by combining model output, drifter observations and images.

APPROACH

I develop theoretical approaches to the data fusion problem in context of the possibility theory (fuzzy logic) and in the framework of the classical theory of random processes and fields covered by stochastic partial differential equations. I design computational algorithms derived from the theoretical findings. A significant part of the algorithm validation is their testing via stochastic simulations. Such an approach provides us with an accurate error analysis. Together with my collaborators from Rosenstiel School of Marine and Atmospheric Research (RSMAS), Consiglio Nazionale delle Ricerche (ISMAR, LaSpezia, Italy), Naval Research Laboratory (Stennis Space Center, Mississippi), ENEA (Rome, Italy), Koc University (Istanbul, Turkey) we implement the algorithms in concrete ocean models such as QG, MICOM , and NCOM as well as carry out statistical analysis of real data sets by means of new methods.

WORK COMPLETED

1. Data fusion for estimating surface velocities.

Two efficient fuzzy logic based algorithms of combining tracer observations with a model output were developed and compared. Both methods appeal to the intersection $D = D_m \cap D_o$ of the credibility regions for the estimated velocity vector coming from the model and observations respectively. The first algorithm called GC takes the geometrical center of D as a combined estimate of the unknown velocity, while the second one (CM) addresses the center of mass of D. A theoretical error analysis was provided for the GC algorithm. The procedures were first tested on an idealized model of gyre superposed by a regular eddy structure, and then, in the framework of 'twin' experiment with NCOM at a specific area of the Ligurian Sea. A compatibility measure between tracer data and a model output was suggested and analyzed.

2. Mixing in the presence of shear

A theoretical investigation of the absolute and relative dispersion in stochastic flows with constant drift gradients was conducted. In particular, different regimes for the relative dispersion were identified depending on the type of the mean velocity stagnation points (hyperbolic or elliptic) and on the Hurst exponent characterizing the dynamics type (local or non-local).

3. Fusion data for estimating RD.

Theoretical relations between relative dispersion and statistical characteristics of a continuously distributed tracer were established to lay ground for data fusion procedures aiming at estimating RD from images. Similar relations were established between the finite size Lyapunov exponent (FSLE) and Eulerian characteristics of the underlying velocity field to estimate the former from tracer snapshots. A fusion algorithm has been developed for estimating RD by combining a model output, drifter data, and images which is based on ideas of fuzzy logic.

RESULTS

1. Principal results in developing and testing the GC and CM data fusion algorithms are as follows.
- GC is computationally simpler, but less stable and less accurate than CM. The difference in stability and accuracy is especially manifested near the boundary of the circulation region of interest.
- The estimation error of GC is in very good agreement with the theory which predicts the improvement with respect to the model output at least 30%.
- Both algorithms demonstrated a high computational efficiency exceeding typical assimilation algorithms in several orders.
- Satisfactory estimation results can be reached for the interval shorter than 6 hours between consecutive snapshots. For longer intervals the error goes out of control.
- The data/model compatibility measure μ introduced as a complementary normalized distance between the credibility regions D_m and D_o turned out to be greatly helpful in interpreting estimation results. In particular, small values of μ in a particular place alarm on the model credibility in this place.

Fig.1 illustrates the CM performance in a 'twin' experiment with NCOM in a particular area of the Ligurian sea. Two outputs for May of 2007 separated by time interval of 7 days, were considered as a model and 'real' velocity field. The main feature of the 'real' circulation (first panel) comparatively to the model (second panel) is an intensive stream in the right low corner. The relative model error, computed by averaging over the whole region, was $\sigma_b = 0.5361$. We added the 'unknown' forcing to the tracer advection equation with known amplitude and used the tracer observations initially concentrated in a compact area to estimate the velocity. The CM estimate turns out to be pretty accurate (third panel) and fairly captures the right-low-corner current with the estimate error $\sigma_e = 0.4713$.

The last panel illustrates the compatibility measure for this experiment. As one can see the compatibility is pretty low in the right-low corner which is due to the strong 'real' stream in this area which is not captured by the model. So, it is not surprising that the estimate detects only little fragments of the stream. Another area of low compatibility along the northern and eastern boundaries might be related to artificial tracer boundary conditions in use.

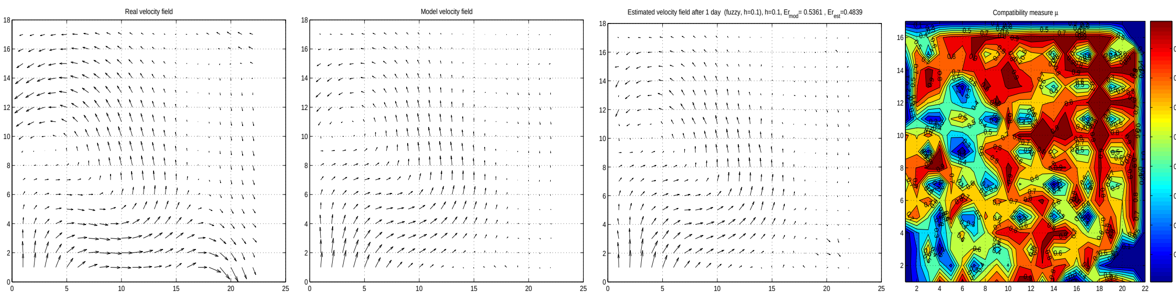


Figure 1. Ligurian Sea example from left to right: 1) 'Real' velocity. 2) Model velocity, $\sigma_b = 0.5361$***. 3) Estimated velocity,*** $\sigma_e = 0.4713$***. 4) Compatibility map of model and 'data'***

2. Concerning with investigation of AD and RD for stochastic flows with shear, we addressed a new LSM developed by PI with the drift given by

$$\mathbf{U} = \mathbf{G}\mathbf{x}, \quad \mathbf{G} = \begin{pmatrix} \gamma & \Omega \\ -\Omega & -\gamma \end{pmatrix}$$

where $\Omega, \gamma > 0$ are vorticity and stretching parameters respectively. The main results are the following.

- It was proven that the inertial regime for both, RD and AD, exists if and only if the stagnation point $\mathbf{x} = \mathbf{0}$ is of elliptic type, i.e. $\Omega > \gamma$ and the Lgrangian correlation time is small enough $\tau < 1/\gamma$.
- An exact expression for the absolute diffusivity tensor was found ant it was shown that its anisotropy exactly copies the anisotropy of the mean drift. The fluctuation parameters, such as the Lagrangian correlation time, velocity fluctuation variance, and the spin [3] affect only the magnitude of the diffusion.

As for RD we mostly concentrated on the intermediate stage $\tau \ll t \ll T$, where T is the particle pair separation time, and assumed a fast enough decay of the energy spectrum (Batchelor regime, [4]).

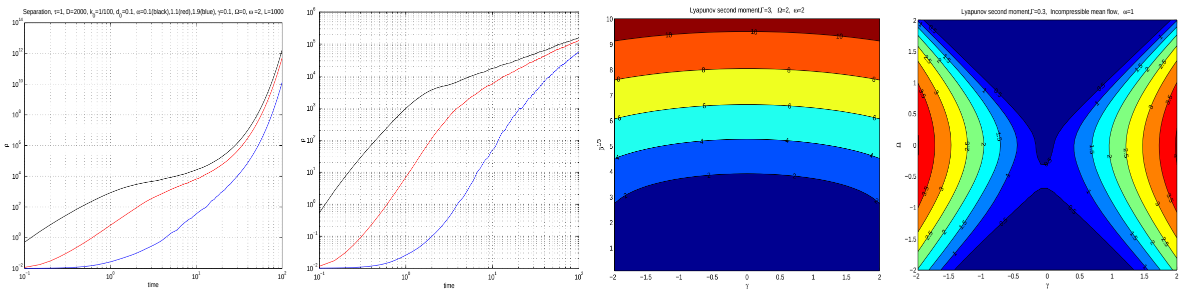
- The well known exponential behavior of RD under the local dynamics [5] was established in the presence of the elliptic drift [6] similarly to the case of zero drift [7]

$$\mathbf{D}(t) \sim \mathbf{K}e^{\Lambda t}$$

where $\mathbf{D}(t)$ is the relative dispersion tensor and Λ is the second Lyapunov exponent.

- We have shown that the anisotropy of $\mathbf{K}$ is determined by both factors, gradients of the mean flow and statistics of the velocity fluctuations. The crucial impact on the mixing ellipse orientation is produced by the normal component of the covariance. The growth rate Λ depends on the drift and fluctuation characteristics as well and can be partly investigated analytically.

In Fig.2 we give examples of the RD curves for both local and non-local dynamics in a vicinity of both hyperbolic ($\Omega < \gamma$, first panel) and elliptic (second panel) stagnation points as well as the dependence of Λ on the shear parameters Ω, γ (third panel) and on β, γ (fourth panel), where β is the characteristic wavenumber of the velocity fluctuations, i.e. $\beta = 1/d$ with d standing for the space correlation scale of the velocity.



***F*igure 2. Relative dispersion in shear flows from left to right: 1) RD vs time and different values of Hurst exponent $h = 0.1$ (black), $h = 1$ (red), $h = 2$ (blue) $\gamma = 0.1, \Omega = 0$, 2) Same for $\gamma = 0.1, \Omega = 2$). 3) RD vs Ω and γ. 4)RD vs β and γ for $\Omega = 2$.**

3. The following result is in the base of the derived fusion algorithm illustrated in Fig.3 for estimating the relative diffusivity (RD) by combining a model output, tracer and drifter obsevations.

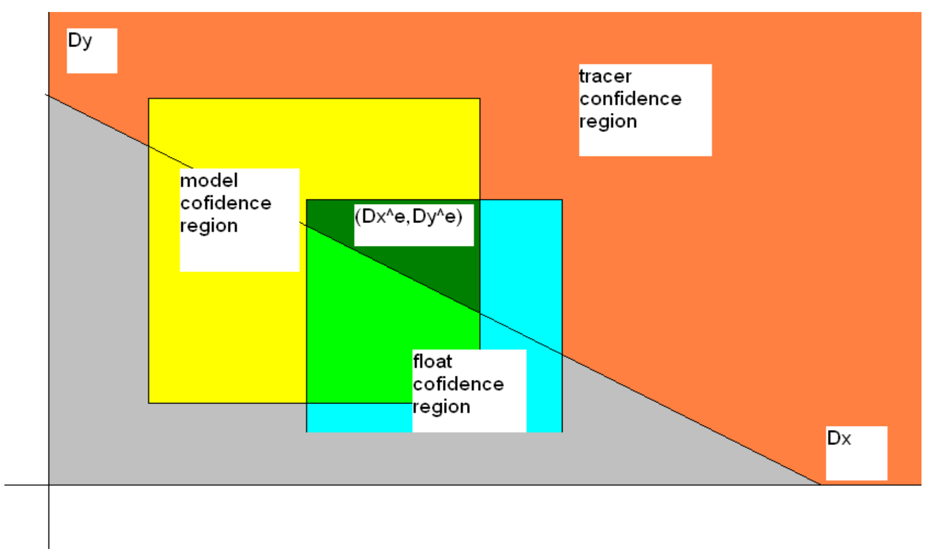


Figure 3: Schematic illustration of the fusion method for the RD estimate. Confidence regions for (D_x, D_y) defined from model experiments (yellow), drifter estimate (green) and tracer estimate (orange) have the common area (dark green) and the combined estimate is its center of mass

- Under wide conditions we proved that for small enough initial separation x_0, y_0

$$\frac{D_x}{d_x^2} + \frac{D_y}{d_y^2} \geq \frac{x_0^2 \langle \theta_x^2 \rangle + y_0^2 \langle \theta_y^2 \rangle}{\sigma_\theta^2} \tag{1}$$

In (1) D_x and D_y are the RD zonal and meridianal components respectively, σ_θ^2 is the variance of the tracer $\theta(t, x, y)$, θ_x, θ_y its gradients, and d_x, d_y are its correlation scales in direction x and y respectively at $t = 0$, the dependence of $D_x, D_y, \langle \theta_x^2 \rangle, \langle \theta_y^2 \rangle$ on t is meant.
Relation (1) determines the orange confidence region. The center of mass $(\hat{D}_x, \hat{D}_y)$ of the intersection (dark green) of the three confidence regions is taken as a combined estimate of (D_x, D_y). Explicit expressions for estimates $(\hat{D}_x, \hat{D}_y)$ were obtained as well.

IMPACT/APPLICATIONS

1. The suggested data fusion procedures for estimating surface velocities and relative dispersion from images, drifters, and a model output could be a supplement to existing assimilation algorithms explicitly involving OGCM.
2. The developed theory of the dispersion in stochastic flows leads to better understanding turbulent mixing in the presence of shear flow

TRANSITIONS

The developed velocity fusion algorithm was used in RSMAS to test it in NCOM circulation model. It is planned to apply the method to real data in the Mediterranean by the same RSMAS group in collaboration with ISAC-CNR remote sensing group (Rome, Italy).

RELATED PROJECTS

1. "Predictability of Particle Trajectories in the Ocean", ONR, PI T.Ozgokmen , RSMAS, N00014-05-1-0095
2. "Lagrangian turbulence and transport in semi-enclosed basins and coastal regions", ONR, PI A Griffa, RSMAS, N00014-05-1-0094

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